GCE

Mathematics A

H240/01: Pure Mathematics

A Level

Mark Scheme for June 2022

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2022

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

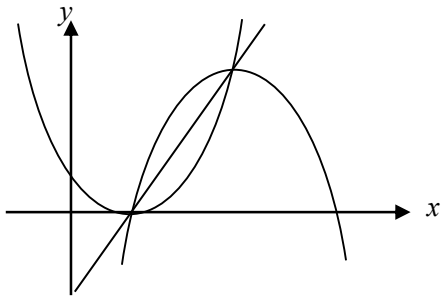
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

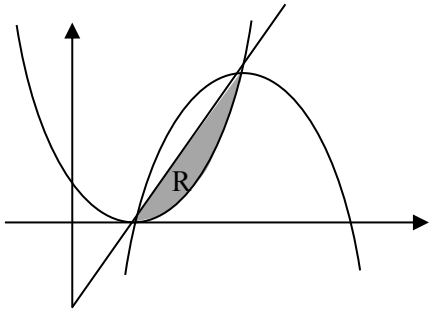
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$0.5\times0.5\left\{0+2\sqrt{2}+2\left(\frac{\sqrt{5}}{2}+\sqrt{3}+\frac{\sqrt{21}}{2}\right)\right\}$	**B1**	**1.1a**	State the 4 correct non-zero y-values and no others	Exact values (including unsimplified) or decimal equivs (0, 1.12, 1.73, 2.29, 2.83) – 3sf or better B0 if other ordinates seen unless clearly not intended to be used
				M1*	**1.1a**	Attempt to find area between x = 1 and x = 3, using $k\{y_0 + y_n + 2(y_1 + ... + y_{n-1})\}$	Big brackets need to be seen or implied y-values must be correctly placed Must be using attempts for at least 4 y-values (but no need to see $y = 0$ explicitly) Condone using other than 4 intervals as long as values equally spaced between x =1 and x = 3
				M1d*	**1.1a**	Use $k = 0.5 \times 0.5$ soi	Dep on previous M1 Or using $k = 0.5h$, h consistent with their different number of intervals
			= 3.28	**A1**	**1.1**	Obtain 3.28, or better	Allow answers to > 3sf, as long as they round to 3.28
				[4]			
1	(b)		Under-estimate, as the tops of the trapezia are below the curve	**B1**	**3.2b**	Under-estimate, with any valid explanation	Condone just 'trapezia under curve' Or curve is concave / decreasing gradient (not decreasing function) Accept explanation on diagrams Allow comparing to true value (3.36) B0 if any additional incorrect or contradictory statements
				[1]			

Question			Answer	Marks	AO	Guidance	
1	(c)		Use more trapezia, of a lesser width, between the same limits	B1 [1]	3.2b	Convincing reason	Condone just 'more trapezia' or 'narrower trapezia' Could refer to strips or intervals
2	(a)		eg $1 > -2$, but $1^2 < (-2)^2$ as $1 < 4$	B1 [1]	2.1	Any correct counterexample, and contradiction identified	Initial inequality soi and then contradiction eg $-3 > -4$ but $9 < 16$ (or $9 \not> 16$)
2	(b)	(i)	eg $\sin 150^\circ = 0.5$ as well	B1 [1]	2.3	Any correct statement	Identifies that $\sin x = 0.5$ could give values of x other than 30° Either specific example or general statement eg 'many to one' function
2	(b)	(ii)	$\sin x^\circ = 0.5 \Leftarrow x^\circ = 30^\circ$	B1 [1]	2.5	Any correct relationship	If attempting to write general solution then must be fully correct eg $x = 30^\circ + 360n^\circ$, $x = 150^\circ + 360n^\circ$ Condone $\leftarrow$ instead of $\Leftarrow$
2	(c)		$(4n) + (4n + 4) + (4n + 8) + (4n + 12)$, where n is an integer	B1*	2.1	Four consecutive multiples of 4 written correctly in terms of n, or any other variable	Allow BOD if n not explicitly stated to be an integer Sufficient to just list the 4 terms, rather than as a sum Not necessarily starting on $4n$ Could also define k as a multiple of 4 and then have k, $k + 4$ etc
			$= 16n + 24$ $= 8(2n + 3)$	M1 dep*	2.1	Correctly sum terms, and correctly take out common factor of 8	Or sum and then consider each term separately Could be a different factor if using k

Question			Answer	Marks	AO	Guidance	
			$2n+3$ is an integer, so $8(2n + 3)$ is a multiple of 8	**A1** **[3]**	**2.4**	Conclude appropriately	Allow BOD if $2n + 3$ not explicitly stated to be an integer If using k... expect $8(0.5k + 3)$ then justify $0.5k$ as an integer, or $4(k + 6)$ then justify $k + 6$ is a multiple of 2
3	**(a)**		**DR** $2x^2 - 8x + 6 = 0$ $x^2 - 4x + 3 = 0$	**M1**	**1.1**	Equate, and rearrange to three term quadratic	Attempt to gather like terms, but not necessarily on same side of equation Condone no '= 0'
			$(x - 1)(x - 3) = 0$	**M1**	**1.1**	Attempt to solve quadratic	If factorising then expansion should give x^2 and one other term correct Quadratic formula should be correct – allow one slip when substituting as long as general formula already seen as correct Completing the square needs to go as far as $x - p = \pm\sqrt{q}$
			$x = 1$, $x = 3$	**A1**	**1.1**	Obtain both correct x values	Or one correct (x, y) coordinate following a correct factorisation oe
			(1, 0) and (3, 4)	**A1**	**1.1**	Obtain both correct pairs of coordinates	Allow as eg $x = 1$, $y = 0$ as long as pairings are clear

Question			Answer	Marks	AO	Guidance	
				[4]			**SC** If no method shown for solving quadratic then allow **M1** for obtaining 3 term quadratic **A1** for $x = 1$, $x = 3$ **A1** for (1, 0) and (3, 4) **SC** If no method at all shown then allow **B1** for both (1, 0) and (3, 4)
3	**(b)**			**M1**	**1.1**	Attempt graph of $y = 2x - 2$, with positive gradient and negative intercept	No need for line to actually intersect with negative y-axis as long as it goes beneath positive x-axis
				A1	**1.1**	Graph of $y = 2x - 2$ passing through both points of intersection of the two quadratic graphs	Must pass through both points
				[2]			

Question			Answer	Marks	AO	Guidance	
3	(c)			B1FT	2.2a	Correct region labelled with R, or otherwise clearly identified	FT any straight line that splits the overlap area into two finite regions, with the lower region identified Allow for straight line with negative gradient as well, but not $x = k$
				[1]			

4	(a)		$2(x + 1.5)^2 + 2.5$	B1 B1 B1FT [3]	1.1a 1.1a 1.1a	$p = 2$ $q = 1.5$ $r = 2.5$	Could be implied by $2(x + q)^2 + r$ Could be implied by $p(x + 1.5)^2 + r$ FT on their p and q ie $7 - pq^2$
4	(b)		$(-1.5, 2.5)$	B1FT B1FT [2]	1.1 1.1	Correct x-coordinate Correct y-coordinate	FT on their (a) Could come from differentiation FT on their (a) No FT on incorrect x-value from differentiation
4	(c)		minimum value of the function = 2.5	B1FT	3.1a	FT on their minimum value	Allow BOD if different answers in **(a)** and **(b)** 2.5 must be stated as, or clearly intended to be, the minimum value Just (... , 2.5) is insufficient

Question			Answer	Marks	AO	Guidance	
			$\tan\theta = -1.5$ $\theta = -56.3^o$	**M1**	**3.1a**	Attempt to solve $\tan\theta =$ their (-1.5)	To obtain numerical value for θ Allow an angle in radians (expect -0.983 rad) Allow BOD if different answers in **(a)** and **(b)**
			$\theta = 124^o$	**A1** **[3]**	**1.1**	Obtain 124^o, or better	**A0** if additional solutions Condone approaches other than 'hence' eg **B1** – attempt to solve $\tan\theta = -1.5$, from correct derivative (expect $4\tan\theta\sec^2\theta + 6\sec^2\theta = 0$) **B1** – obtain $\theta = 124^o$ **B1** – obtain min value of 2.5 (no FT)
5	**(a)**	**(i)**	4 units in the negative x-direction	**M1**	**1.1**	Indicate horizontal translation (in either direction) in some way with magnitude of 4 ('units' not required)	**B1** for $\begin{pmatrix}4\\0\end{pmatrix}$ Condone informal language as long as intent is clear eg 'left' (or even 'right', as either direction allowed) **M0** if ambiguous eg 'in' or 'on' the x-axis
				A1 **[2]**	**2.5**	or 4 in negative x-direction Correct language needed	**B2** for $\begin{pmatrix}-4\\0\end{pmatrix}$ Must now be correct language so **A0** for eg 'along' the x-axis or 'left' Allow 'parallel to the x-axis' or 'horizontal'

Question			Answer	Marks	AO	Guidance	
5	(a)	(ii)	in the y-direction with sf 16	**B1** **B1** **[2]**	**3.1a** **1.1**	Identify direction - correct language needed or 2^4	Allow 'x-axis invariant', 'parallel to the y-axis' or 'vertical' Condone 'positive' y-direction (as given function > 0) 'scale factor' or 'factor' needed (condone 'stretch' factor) Not dep on previous B1, but must have indicated vertical stretch in some way, including informal language such as 'upwards' Cannot be ambiguous language, such as 'in', 'on', 'across' the y-axis
5	(b)		**DR** $\log_2(8x(1-x)) = 1$	**M1**	**1.1a**	Correctly combine two correct log terms	Or $\log_2(8x) = \log_2 \frac{2}{1-x}$ Or $3 + \log_2(x(1-x)) = 1$ Or $\log_2(4x(1-x)) = 0$ **OR** use indices base 2 on both sides (ie $8x = 2^{1-\log_2(1-x)}$) and use rules of indices to split eg $8x = 2 \times 2^{-\log_2(1-x)}$

Question			Answer	Marks	AO	Guidance	
			$8x(1-x) = 2$	**M1**	**1.1a**	Correct method to remove logs	Correctly used on equation of form $\log_2 f(x) = \log_2 g(x)$ or $\log_2 f(x) = k$ **OR** correct method to deal with log term – expect $8x = \frac{2}{1-x}$
			eg $8x^2 - 8x + 2 = 0$ or $8x(1-x) = 2$ or $8x = \frac{2}{1-x}$	**A1**	**1.1**	Any correct equation not involving logarithms	Could still contain brackets and / or fractions
			$x = 0.5$	**A1**	**1.1**	Obtain $x = 0.5$	**A0** if additional solutions **DR** so no credit for answer only
				[4]			

6	**(a)**		$3^5 + 5\times3^4\times(2x) = 243 + 810x$	**B1**	**1.1**	Obtain $243 + 810x$	Condone $3^5 + 810x$ Allow terms not written as a sum eg written separately, or linked with a comma
			$10\times3^3\times(2x)^2$ or $10\times3^2\times(2x)^3$	**M1**	**1.1a**	Attempt at least one further term – product of correct binomial coeff, power of 3 and attempted power of $2x$, with powers totalling 5	Binomial coeff must be numerical; ${}^5C_2$ is not yet enough Allow BOD if brackets missing when index is applied to $2x$, even if never recovered eg $540x^2$ or $180x^3$
			$+ 1080x^2$	**A1**	**1.1**	Obtain correct third term	Coefficient simplified Terms separate, listed or summed

Question			Answer	Marks	AO	Guidance	
			$+ 720x^3$	**A1**	**1.1**	Obtain correct fourth term	Coefficient simplified Could be separate term, part of a list or part of a sum If expanding brackets then mark as above, but all 5 sets of brackets must be considered (allow irrelevant terms to be discarded)
			Alternative method			**Expanding** $\left[3\left(1+\frac{2}{3}x\right)\right]^5$	
			$243 + 810x$ or $243\left(1+\frac{10}{3}x\right)$	**B1**		First two terms correct	Allow with 243 still outside the bracket
			$243\left(\frac{40}{9}x^2\right)$ or $243\left(\frac{80}{27}x^3\right)$	**M1**		Attempt one further term	Condone just 3 not 3^5 being used, but must be the correct binomial coeff and an attempt at the correct power of $\frac{2}{3}x$, but allow BOD if no brackets
				A1		Either 3rd or 4th term correct	Allow with 243 still outside the bracket
			$243 + 810x + 1080x^2 + 720x^3$	**A1**		Fully correct expansion	With the 243 now multiplied into the expansion
				[4]			
6	**(b)**		$x = y + 2y^2$	**B1**	**3.1a**	Identify correct substitution	Could be stated, or implied by use in their binomial expansion
			$1080(y + 2y^2)^2 + 720(y + 2y^2)^3$	**M1**	**1.1a**	Attempt to use binomial from **(a)** with their 2 term substitution	Must substitute into at least the x^2 and x^3 terms from their **(a)** Allow **M1** if using $2y + 4y^2$ as their substitution
			$4320y^3 + 720y^3$	**M1**	**1.1a**	Attempt expansion to obtain the two relevant terms in y^3	**M0** if any other y^3 terms Expect 4(their 1080) and (their 720) Allow **M1** if using $2y + 4y^2$ as their substitution - expect 16(their 1080) and 8(their 720)

Question			Answer	Marks	AO	Guidance	
			coeff of y^3 is 5040	**A1**	**1.1**	Allow $5040y^3$	Ignore any other non-cubic terms
			Alternative method 1			**Attempting binomial expansion of $(3 + (2y + 4y^2))^5$ or $((3 + 2y) + 4y^2)^5$**	
			eg $((3 + 2y) + (4y^2))$	**B1**		Group into two expressions, and attempt to use them	
			eg $(3 + 2y)^5 + 5(3 + 2y)^4(4y^2)$	**M1**		Use their groups to obtain the appropriate two elements of their binomial expansion (ie those that would give y^3 terms)	
			eg $(\ldots + 720y^3 + \ldots) + 5(\ldots 4.3^3.2y\ldots)(4y^2)$ $= 720y^3 + 4320y^3$	**M1**		Expand to attempt the two y^3 terms, and no others	
			coeff of y^3 is 5040	**A1**		Obtain 5040	
			Alternative method 2			**Attempting to expand all 5 brackets**	
			eg $(3 + 2y + 4y^2)^5 =$ $(81 + 216y + 648y^2 + 960y^3 \ldots)(3 + 2y + 4y^2)$	**M1**		Attempt to use all 5 brackets	An attempt to use all 5 is sufficient
			$(216y \times 4y^2) + (648y^2 \times 2y) + (960y^3 \times 3)$	**M1**		Attempt all products that would give a y-cubed term	Condone additional terms, even those that would give another y^3 term Irrelevant terms (ie powers greater than 3) may never be seen

Question			Answer	Marks	AO	Guidance	
			$864y^3 + 1296y^3 + 2880y^3$	**A1**		Obtain correct terms or coefficients, with no more than one incorrect	They must have attempted all of the expected y^3 terms, and no more, with no more than one coefficient error If $(3 + 2y + 4y^2)^4 \times (3 + 2y + 4y^2)$ then expect 2880 + 1296 + 864, If $(3 + 2y + 4y^2)^3 \times (3 + 2y + 4y^2)^2$ then expect 1368 + 1728 + 1512 + 432 If they have not yet combined like terms then this A mark can only be implied by a later correct answer or relevant correct combination of terms
			coeff of y^3 is 5040	**A1**		Obtain 5040	
				[4]			

7			$6x^2 + 6y + 6x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$	**M1**	**1.1a**	Attempt implicit differentiation	Either of the two $\frac{dy}{dx}$ terms correct, allowing sign errors Condone $6x^2dx + 6ydx + 6xdy - 6ydy$
				B1	**1.1a**	Use product rule correctly on middle term	Both terms correct Must now be $6y + 6x\frac{dy}{dx}$, or implied in a correct expression for $\frac{dy}{dx}$

Question			Answer	Marks	AO	Guidance	
				A1	**1.1**	Obtain correct derivative on LHS	Condone missing or incorrect RHS Must now have $\frac{dy}{dx}$ and not just dy or dx in terms
			$6x^2 + 6y + 6x - 6y = 0$	**M1**	**3.1a**	Use $\frac{dy}{dx} = 1$ in their equation	Must now be equation, but RHS could be incorrect (eg '= 2')
			$x^2 + x = 0$ $x = 0, x = -1$	**B1**	**1.1a**	Solve correct quadratic in x to obtain two correct roots (possibly **BC**) Quadratic must come from correct implicit differentiation	**B0** if x 'cancelled' in quadratic to give $x = -1$ as only root, but **M1A1** still available
			$x = 0$ gives $3y^2 = -2$, but y^2 has to be ≥ 0, so no solutions	**B1**	**2.3**	Explicitly reject $x = 0$, with reasoning $x = 0$ must come from $x^2 + x = 0$	eg negative numbers cannot be square rooted or $y^2 \neq -\frac{2}{3}$ 2 as y is real (just $y^2 \neq -\frac{2}{3}$ is insufficient) Must be sensible reason and not just 'math error' or 'not possible' Could say that there are only imaginary (or not real) roots – condone 'complex' roots
			$x = -1$ gives $3y^2 + 6y + 4 = 0$ $b^2 - 4ac = 36 - 48 = -12$	**M1**	**2.1**	Attempt to determine the number of real roots of their 3 term quadratic in y	From substituting their x value into the equation of the curve Consider discriminant, or use quadratic formula, or attempt minimum value of function
			$-12 < 0$ hence no (real) roots	**A1** **[8]**	**2.4**	Obtain correct discriminant from correct quadratic and conclude appropriately $x = -1$ must come from $x^2 + x = 0$	If using quadratic formula then it must be fully correct and attention drawn to why there are no real roots

Question	Answer	Marks	AO	Guidance

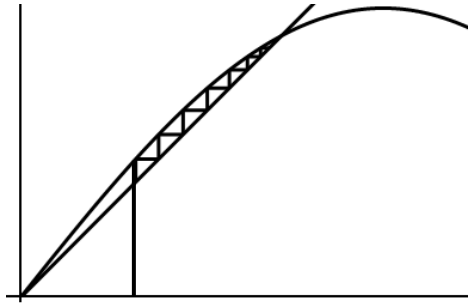
8	**(a)**		20 (minutes) 97.2 (grams)	**B1** **B1** **[2]**	**3.3** **3.4**	Obtain $t = 20$ Obtain $m = 97.2$	Allow [19.9, 20.1] from setting up and using exponential model Allow [97.1, 97.3] from setting up and using exponential model
8	**(b)**	**(i)**	$160e^{-0.055t} = 80$	**B1**	**3.4**	Equate given model to 80	soi, so could be $e^{-0.055t} = 0.5$

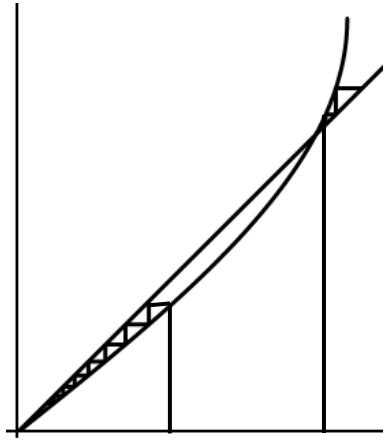
Question			Answer	Marks	AO	Guidance	
			$e^{-0.055t} = 0.5$ $-0.055t = \ln 0.5$	**M1**	**3.4**	Attempt correct process to find value of t, as far as dealing with exponential term	Rearrange to $e^{-0.055t} = k$, and hence obtain $-0.055t = \ln k$ If introducing logs straight away then need to get as far as $\ln 160 - 0.055t = \ln$(their 80)
			$t = 12.6$ (minutes)	**A1** **[3]**	**1.1**	Obtain $t = 12.6$, or better	If more sig fig given, then allow answers which round to 12.60 (the more accurate answer is 12.602676..)
8	**(b)**	**(ii)**	$\frac{dm}{dt} = -8.8e^{-0.055t}$	**B1**	**3.4**	Correct derivative soi	Allow unsimplified No need to see $\frac{dm}{dt} =$
			$-8.8e^{-0.055 \times 15}$	**M1**	**3.4**	Substitute $t = 15$ into their derivative	Must be of the form $ke^{-0.055t}$, with $k \neq 160$ Possibly still with k unsimplified Substitution sufficient, no need to evaluate for M1
			$= -3.86$, hence rate of decay is 3.86 grams/minute	**A1** **[3]**	**1.1**	Units required, and positive answer	Must follow correct derivative ie negative coefficient No need to see –3.86 first, but A0 if clear error Accept 3.9 grams/minute Accept g/m for grams/minute

Question			Answer	Marks	AO	Guidance	
8	(c)		For A, $\frac{dm}{dt} = -63.9e^{-0.0511t}$ Rate of decrease at $t = 15$ is 29.7 g/min hence A decaying at a faster rate	B1	3.4	State A, with clear comparison	Insufficient to just say that A has a greater initial mass – needs to consider decay factor as well Allow solutions that identify that B is decaying faster, with supporting evidence eg after 10 minutes, B's mass is 92.3g which is 58% of initial mass whereas A is 60% of initial mass so B decaying faster eg A's half-life is 13.6 so B is decaying faster eg change initial mass in model B to 1250 then when $t = 10$ B's mass would be 721g which is less than 750 hence decaying faster eg compare coefficients of t (for A, coeff is – 0.0511); B's is of a greater magnitude hence decaying faster For either solution, the conclusion and the supporting evidence must be consistent Numerical supporting evidence must be correct, allowing for slight inaccuracies from using different numbers of sig fig (see appendix)

Question			Answer	Marks	AO	Guidance	
				[1]			
9			$dx = 2\cos\theta d\theta$	M1	1.1a	Attempt to link dx and $d\theta$	Allow sign error only
			$\int\sqrt{4-x^2}dx = \int\sqrt{4-4\sin^2\theta}.2\cos\theta d\theta$	M1	3.1a	Attempt to write integrand in terms of θ	Must substitute for both function and dx Can follow M0 but do not allow just $dx = d\theta$
			$= \int\sqrt{4\cos^2\theta}.2\cos\theta d\theta$	A1	1.1	Obtain correct integrand in terms of $\cos\theta$ only	Condone no $d\theta$, as long as previously seen
			$= \int 4\cos^2\theta d\theta$				
			$= \int(2\cos 2\theta + 2)d\theta$	M1	2.1	Attempt use of double angle formula	Using $\cos 2\theta = \pm 2\cos^2\theta \pm 1$ Integrand must be of form $k\cos^2\theta$, which must have come from correct method with coefficient errors only
			$= \sin 2\theta + 2\theta$	A1FT	1.1	Integrate to obtain $\sin 2\theta + 2\theta$	FT on $a\cos 2\theta + b$ only
			$\left[\sin 2\theta + 2\theta\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = \left(\sin\frac{2}{3}\pi + \frac{2}{3}\pi\right) - \left(\sin\frac{2}{6}\pi + \frac{2}{6}\pi\right)$ $= \left(\frac{1}{2}\sqrt{3} + \frac{2}{3}\pi\right) - \left(\frac{1}{2}\sqrt{3} + \frac{1}{3}\pi\right)$	M1	2.1	Attempt use of limits	Must be correct limits (either x or θ, as long as consistent with their integral), correct order and subtraction Allow M1 for use of limits in any integration attempt in terms of θ Allow M1 for either expressions that still involve sin, or exact equivs M0 for decimal values, even if then stated to be the same as $\frac{1}{3}\pi$ Condone eg $\frac{1}{2}\sqrt{3}$ from $\sin 120^\circ$, but M0 if degrees used in linear term
			$= \frac{1}{3}\pi$ **A.G.**	A1	2.1	Obtain given answer of $\frac{1}{3}\pi$	Must see both surd values, or an explanation as to why $\sin\frac{2}{3}\pi = \sin\frac{2}{6}\pi$
				[7]			

Question			Answer	Marks	AO	Guidance	
10	(a)		area $OMB = \frac{1}{2}\left(\frac{1}{2}r\right)r\sin\theta$	B1	1.1	Correct (possibly unsimplified) area of OMB	Could use other than r for the radius Could set their variable equal to OM, giving a radius that is double this eg $OM = x$ so area = $x^2\sin\theta$
			$2\left(\frac{1}{2}r^2\theta - \frac{1}{4}r^2\sin\theta\right) = 3\left(\frac{1}{4}r^2\sin\theta\right)$ OR $2\left(\frac{1}{2}r^2\theta\right) = 5\left(\frac{1}{4}r^2\sin\theta\right)$ OR $3\left(\frac{1}{2}r^2\theta\right) = 5\left(\frac{1}{2}r^2\theta - \frac{1}{4}r^2\sin\theta\right)$	M1	3.1a	Attempt to use ratio on two correct areas	Using two of OMB ($\frac{1}{4}r^2\sin\theta$), MAB $\left(\frac{1}{2}r^2\theta - \frac{1}{4}r^2\sin\theta\right)$ and OAB ($\frac{1}{2}r^2\theta$) oe with their variable Must be two correct areas Must be using the correct ratio for their two areas ie 2:3 if using OMB and MAB, 2:5 if using OMB and OAB or 3:5 if using MAB and OAB Allow ratio to be used the wrong way around eg $2OMB = 3MAB$
				A1	2.1	Correct equation, in two variables (ie θ and their r)	Any correct statement linking the two areas Could use other than r for the radius Or $2x^2\theta - x^2\sin\theta$
			$\theta - \frac{1}{2}\sin\theta = \frac{3}{4}\sin\theta$ $\theta = 1.25\sin\theta$ **A.G.**	A1 [4]	2.1	Simplify to given answer	At least one line of working once ratio used

Question			Answer	Marks	AO	Guidance	
10	(b)		0.599	B1	1.1a	Obtain correct first iterate	3sf or better – more accurate answer is 0.599281923... Condone truncating if more sig fig given
			0.705, 0.810	M1	1.1a	Attempt correct iterative process to find at least 2 more values	M1 is for the correct process for finding θ_3 and θ_4, but these may be incorrect M0 if working in degrees
			root = 1.13	A1	1.1	Obtain 1.13	Possibly following B0 if first iterate is wrong but process then self corrects Must follow M1 ie a clear attempt to use the correct iterative process Must be 3sf Once M1 is awarded, allow A1 for 1.13 even if an incorrect iterate seen, as process will recover
				[3]			
10	(c)			B1*	3.1a	Draw $y = \theta$ on diagram	Draw straight line, starting at the origin which intersects the graph Allow point of intersection to be greater than $\theta = \frac{1}{2}\pi$ Ignore incorrect labels, such as $y = x$

Question			Answer	Marks	AO	Guidance	
				B1 dep*	**2.1**	Draw correct iterative process on diagram	Vertically into the curve, then horizontally into the straight line, as far as the root Initial value should be before root Needs point of intersection to be before $\theta = \frac{1}{2}\pi$
				B1	**1.2**	State 'staircase' convergence	Mark independently from other parts of question, including an incorrect diagram, as staircase can be deduced from the iterates in **(b)**
				[3]			
10	**(d)**			**B1***	**3.1a**	Draw graph of $y = \sin^{-1}0.8\theta$, for $\theta \geq 0$	Just need correct shape for $y = \sin^{-1}k\theta$ graph – a one to one function that starts at the origin (ignore any $\theta < 0$) and has increasing gradient for all θ
				B1 dep*	**3.2a**	Draw $y = \theta$, and show staircase divergence from the root found in **(b)**, on at least one side of the root	Straight line from the origin to intersect their graph Diagram is sufficient for **B1** – no comment or explanation required
				[2]			

Question	Answer	Marks	AO	Guidance
11 (a)	$\int e^{3y}dy = \int 3x^2 \ln x dx$	M1	3.1a	Separate variables and attempt integration of at least one side Allow ke^{3y}, with $k \neq 1$, as 'attempt' at integration of LHS 'Attempt' at RHS may not be use of integration by parts Allow BOD on missing integral sign / missing dy / missing dx as long as intention clear
	$\int e^{3y}dy = \frac{1}{3}e^{3y}$	B1	1.1	Correct LHS B0 if still part of an expression also involving x
	$\int 3x^2 \ln x dx = x^3 \ln x - \int x^2 dx$	M1	3.1a	Attempt integration by parts on RHS – must have correct parts As far as attempt at $x^3 \ln x - \int x^2 dx$, possibly with $\int \frac{1}{x} x^3 dx$ not yet simplified
	$= x^3 \ln x - \frac{1}{3}x^3 + c$	A1	1.1	Correct RHS (condone no + c) Condone no modulus sign on ln$x$
	$\frac{1}{3}e^3 = e^3 \ln e - \frac{1}{3}e^3 + c$ so $c = -\frac{1}{3}e^3$	M1	1.1a	Attempt use of (e, 1) to find c Used in an equation involving x, y and c, following some integration attempt of both sides As far as finding c, either exact or as a decimal M1 can be implied by sight of $-\frac{1}{3}e^3$ or –6.695… following a correct equation
	$\frac{1}{3}e^{3y} = x^3 \ln x - \frac{1}{3}x^3 - \frac{1}{3}e^3$ $e^{3y} = 3x^3 \ln x - x^3 - e^3$	A1	1.1	Obtain correct equation, in required form Any equivalent form on the RHS, but must be $e^{3y} = ...$ A0 if decimal approximation for e^3

Question			Answer	Marks	AO	Guidance	
				[6]			
11	**(b)**		$e^{3y} = 3e^6\ln e^2 - e^6 - e^3$ $= 6e^6 - e^6 - e^3$ $= 5e^6 - e^3$	**M1***	**2.1**	Substitute $x = e^2$, into their integral involving $\ln x$, and attempt to simplify	$\ln x$ may be $\ln x^p$ if any coefficient has been taken into the ln term As far as correctly simplifying the ln term to remove ln Must be working exactly, so M0 if decimals seen before ln dealt with
			$3y = \ln(e^3(5e^3 - 1))$ $= 3 + \ln(5e^3 - 1)$	**M1 dep***	**2.1**	Introduce logs correctly, and attempt to rearrange to given form	Their equation must have two terms, or possibly more, with the terms having a common factor of e^k Attempt must go as far as splitting into the sum of two terms, with $\ln e^k$ simplified to k
			$y = 1 + \frac{1}{3}\ln(5e^3 - 1)$	**A1** **[3]**	**2.1**	Obtain $y = 1 + \frac{1}{3}\ln(5e^3 - 1)$	No need to state a, b and c explicitly

Question			Answer	Marks	AO	Guidance	
12	(a)		$\frac{dx}{dt} = \frac{-1}{t^2}$, $\frac{dy}{dt} = 2$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	1.1a	Attempt correct process to find gradient in terms of t or p	Correctly combine attempts at two derivatives Need $\frac{dx}{dt} = kt^{-2}$ and $\frac{dy}{dt} = 2$ **SC B1** for gradient of $-2x^{-2}$ if it is never seen in terms of t or p
			$\frac{dy}{dx} = -2t^2$	A1	2.1	Obtain correct gradient	In terms of t or p
			$y - 2p = -2p^2\left(x - \frac{1}{p}\right)$	M1	1.1a	Attempt equation of tangent	Condone still working in terms of t Allow mixture of t and p as long as convincingly recovered Using their gradient from a differentiation attempt, but not dependent on first M1 Substitution into $y - y_1 = m(x - x_1)$ or equation involving c from $y = mx + c$
			$y = -2p^2x + 4p$ **A.G.**	A1	2.1	Obtain given answer	Must now be in terms of p Expand brackets and simplify to given answer, or find c and substitute back into equation
				[4]			

Question			Answer	Marks	AO	Guidance	
12	(b)		$m' = \frac{1}{2p^2}$	B1FT	1.1a	Correct (unsimplified) gradient of normal, following their derivative	Gradient in terms of t or p, but not x Could either FT on their incorrect derivative or deduce the gradient from the equation given in (a)
			$y - 2p = \frac{1}{2p^2}\left(x - \frac{1}{p}\right)$ $y = \frac{1}{2p^2}x + 2p - \frac{1}{2p^3}$	M1	1.1	Attempt equation of normal	Attempt to use their gradient and P Allow mixture of t and p as long as convincingly recovered Substitution into $y - y_1 = m(x - x_1)$ or equation involving c from $y = mx + c$
				M1	3.1a	Use $y = 0$ to attempt x-coordinate of B	Using their attempt at normal equation As far as finding an expression for x
			at B, $y = 0$ so $x = 2p^2\left(\frac{1}{2p^3} - 2p\right) = \frac{1}{p} - 4p^3$	A1	2.1	Correct x-coordinate for B	Any equivalent form
			at A, $y = 0$ so $x = \frac{4p}{2p^2} = \frac{2}{p}$	B1	2.1	Correct x-coordinate for A	Any equivalent form
			$PA = \sqrt{\left(\frac{1}{p}\right)^2 + (2p)^2}$ $PB = \sqrt{\left(4p^3\right)^2 + (2p)^2}$	M1	3.1a	Attempt length of PA or PB	Or **M1** for attempting one of $(PA)^2$ or $(PB)^2$ Must correct distance formula Using the given P, and their coordinates for A and/or B, which must involve a function of p
				A1	2.1	Correct PA and PB	Or correct $(PA)^2$ and $(PB)^2$
			$PA : PB = \frac{1}{p}\sqrt{4p^4 + 1} : 2p\sqrt{4p^4 + 1}$ $= \frac{1}{p} : 2p$ $= 1 : 2p^2$ **A.G.**	A1	2.1	Simplify ratio to obtain given answer	Must show clear method, such as same expression in each square root before cancelling Could also consider fraction and then cancel to deduce given ratio Could simplify $(PA)^2 : (PB)^2$, and then square root to obtain ratio
				[8]			

APPENDIX

Supporting Evidence for Q8(c)

When comparing % remaining or percentage lost in t minutes. Substance B is shown to be decreasing at a faster rate.

Choose t = 15

Substance A

time	0	10	20	50	15
mass (exact)	1250	750	450	97.200	580.948
mass (k=-0.0511)	1250	750	450	97.115	580.796
mass (k=-0.051)	1250	750	450	97.602	581.667

Percentage decreased at t =	0	10	20	50	15
Exact k value	0%	40%	64%	92.22%	53.52%
k=-0.0511	0%	40%	64%	92.23%	53.54%
k=-0.051	0%	40%	64%	92.19%	53.47%

Percentage remaining at t =	0	10	20	50	15
Exact k value	100%	60%	36%	7.78%	46.48%
k=-0.0511	100%	60%	36%	7.77%	46.46%
k=-0.051	100%	60%	36%	7.81%	46.53%

Substance B

time	0	10	20	50	15
mass (exact)	160.00	92.312	53.259	10.228	70.118

Percentage decreased at t =	0	10	20	50	15
Exact k value	0%	42.31%	66.71%	93.61%	56.18%

Percentage remaining at t =	0	10	20	50	15
Exact k value	100%	57.69%	33.29%	6.39%	43.82%

When comparing RATE of decrease at t minutes. Substance A is shown to be decreasing at a faster rate.

Substance A

time	0	10	20	50	15
dm/dt (exact)	-63.853	-38.312	-22.987	-4.965	-29.676
dm/dt (k=-0.0511)	-63.875	-38.318	-22.987	-4.963	-29.679
dm/dt (k=-0.051)	-63.750	-38.282	-22.988	-4.978	-29.665

Substance B

time	0	10	20	50	15
dm/dt (exact)	-8.800	-5.077	-2.929	-0.563	-3.856

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on

01223 553998

Alternatively, you can email us on

support@ocr.org.uk

For more information visit

ocr.org.uk/qualifications/resource-finder

ocr.org.uk

Twitter/ocrexams

/ocrexams

/company/ocr

/ocrexams

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

OCR is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored. © OCR 2022 Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee. Registered in England. Registered office The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA.

Registered company number 3484466. OCR is an exempt charity.

OCR operates academic and vocational qualifications regulated by Ofqual, Qualifications Wales and CCEA as listed in their qualifications registers including A Levels, GCSEs, Cambridge Technicals and Cambridge Nationals.

OCR provides resources to help you deliver our qualifications. These resources do not represent any particular teaching method we expect you to use. We update our resources regularly and aim to make sure content is accurate but please check the OCR website so that you have the most up-to-date version. OCR cannot be held responsible for any errors or omissions in these resources.

Though we make every effort to check our resources, there may be contradictions between published support and the specification, so it is important that you always use information in the latest specification. We indicate any specification changes within the document itself, change the version number and provide a summary of the changes. If you do notice a discrepancy between the specification and a resource, please contact us.

Whether you already offer OCR qualifications, are new to OCR or are thinking about switching, you can request more information using our Expression of Interest form.

Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.